Ants have colonized almost every landmass on Earth. Their success in so many environments has been attributed to their social organization and their ability to modify habitats, tap resources, and defend themselves.
(en.wikipedia.org/wiki/Ant)

Abstract:
It is well known that Modern Portfolio Theory does only work in theory. It requires a reliable estimation of expected returns and the expected covariance matrix (see [1] or any other MPT introduction). The universe of assets must form a multivariate normal distribution. One needs a long time series of historic data to estimate the covariances. But the covariances are not stationary. They change over time.
The Ant Strategy avoids the explicit covariance estimation at all. One starts with an initial population of random portfolios and improves them with Differential Evolution. The optimization criterion is the risk adjusted return of the portfolio itself. The covariance matrix is only implicitly taken into account by the performance of the portfolio.
The behavior of this method is demonstrated for an asset-universe defined in [1] and for the Nasdaq-100. The results are compared to a Momentum approach called the Donkey developed in previous working papers ([2] and [3]). The Ants have an edge over the Donkey.

Revision 1:
Revision 1 adds the Implied-Volatility-Term-Structure (IVTS) as a Stop-Loss signal. The IVTS was developed and successfully applied in previous working papers. This work is directly based on the results in [8]. The IVTS improves the performance further and especially protects effectively against downside risk. Revision 1 is added at the end of the original paper.

The Diversified Portfolio:
In [1] the authors divided the world's major sources of return in 10 asset classes.

<table>
<thead>
<tr>
<th>Asset</th>
<th>ETF</th>
<th>Asset</th>
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</thead>
<tbody>
<tr>
<td>U.S. Stocks</td>
<td>SPY</td>
<td>International REITs</td>
<td>RWX</td>
</tr>
<tr>
<td>European Stocks</td>
<td>VGK</td>
<td>U.S. Intermediate Treasuries</td>
<td>IEF</td>
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<tr>
<td>Japanese Stocks</td>
<td>EWJ</td>
<td>U.S. Long-Term Treasuries</td>
<td>TLT</td>
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<tr>
<td>Emerging Markets Stocks</td>
<td>VWO</td>
<td>Commodities</td>
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<tr>
<td>U.S. REITs</td>
<td>VNQ</td>
<td>Gold</td>
<td>GLD</td>
</tr>
<tr>
<td>U.S. Technology*</td>
<td>QQQ</td>
<td>U.S. High-Yield Bonds*</td>
<td>HYG</td>
</tr>
</tbody>
</table>

Table-1: Diversified Portfolio with basic assets classes
I have added two assets-classes marked with “*” and selected for each class a representative ETF. The Diversified portfolio used in this study consists of these 12 ETFs.

In [1] the authors refine stepwise an equal weighted portfolio to derive what they call an Adaptive Asset Allocation method. The second-last step is similar to the Donkey developed in [2] and [3]. They select the best 5 (the top-half) of their assets and construct from these assets a risk-adjusted portfolio. The weight of each asset is the inverse of it's volatility. The selection is based on the momentum in the last 6 months. The portfolio is rebalanced each month. Unfortunately it is not clear if momentum is the plain return or the return divided by volatility. The Donkey uses the risk adjusted measure.

In the final step the authors claim that they take the covariance of the selected assets into account. This is in contrast to their initial blame of Modern Portfolio Theory. They give no details how this can be done in real trading live. Filippos Zavliaris sent me a mail “Hi, Chrilly, any idea how to solve this question?”. The Ant is the best answer I have found so far.

The Ant:
For the Ant one selects initially 20 random portfolios. The only constraints are: each weight must be in \([0.02,0.25]\) or 0.0. The weights must sum up to 1.0. The lower threshold of 0.02 was selected, because very small weights are a waste of trading effort and costs.

One calculates for each portfolio the daily Sharpe-Ratio. The daily-return benchmark is set to 0.01% or about 2.55% per year. The best window length is 126 trading days or 6 months. This is in full agreement with the momentum-length in [1].

I have tried also the Sortino-Ratio and the Omega-Measure. The performance for these alternative measures is similar.

Note: Post-Modern-Portfolio-Theory uses the Sortino-Ratio and claims to be something quite different. For the Ant it does not really matter.

The random weights are improved with Differential Evolution. One could use any other optimization strategy like threshold-accepting [4], particle-swarm [5] or ant-colony-optimization [6]. Differential Evolution is my favorite tool I have used in all sorts of optimization problems [7]. The optimizer sticks to the weight constraints. The optimizer iterates 20 particles over 50 generations. If one increases the effort e.g. to 50 particles and 100 generations, the performance does not increase any more. One notices even a slight decrease. This is a well known effect in computational learning called over-fitting. The optimizer finds in the historic sample very specific patterns which have no predictive power.

The calculation starts at 2009-12-30 and ends at 2014-03-27. There are 3 major events in this period. The Japanese earthquake in March 2010, the flash-crash in May 2010 and the crash in August 2011. The initial portfolio value is 500.000$. The same initial date and value was used in previous studies [7]. The portfolio is rebalanced at the last calendar day of each month (or the next trading day in the following month).

Graphic-1 shows the performance of the Ant for the Diversified portfolio. The yellow line is the SPY. The overall win is 53.7% for the Ant and 79.2% for the SPY. But the Ant graph is much smoother. The Sharpe-Ratio (calculated from one re-balance date to the next) of the Ant is 1.01 to 0.82 for the SPY. The Ant even gains in the August 2011 crash and has only slight problems in the second half of September 2011. The same holds for the flash-crash. The max. relative drawdown is on 2013-06-20 with 9.47%. The max. relative drawdown of the SPY is 18.61% on 2011-10-03. The monthly Sharpe-Ratio of the Ant is considerable better than any of its constituents. The performance is closest to the High-Yield-Bond ETF HYG (see Graphic-2).
Graphic-1: Ant: Diversified-Portfolio (orange), SPY (yellow) 2009-12-30 till 2014-03-27.

Graphic-2: Ant: Diversified-Portfolio (orange), HYG (yellow) 2009-12-30 till 2014-03-27.
Graphic-3 shows the result for the Donkey-Strategy. One goes the upper half or 6 ETFs long. The momentum is set to 6-months. This is also the best window-length. The overall picture is the same. The final win is 50.3%, the max. relative drawdown is 9.8% at 2013-06-24. The monthly Sharpe-ratio is 0.95. The Ant has a slightly better and smoother behavior. The asset selection criterion is very similar. The difference can be attributed to the fact that the Ant takes implicitly the covariance structure into account. The covariance is ignored in the Donkey. I had in a previous study bad experience with the usual MPT approach. So I decided to omit the relation between the assets at all. The Ant avoids all the fuss of estimating a reliable covariance matrix and takes this information nevertheless into account.
The Nasdaq-100 Ant:
The Nasdaq-100 Ant uses exactly the same method. The universe is formed by the Nasdaq-100 stocks plus the index ETF QQQ. The best window length is now 21 trading-days. One considers only the last month. Graphic-4 shows the result. The overall performance is 232.9% with a max. relative drawdown of 16.7% on 2011-08-19. The Sharpe-Ratio is 1.18.
If one uses like before a window-length of 126 days, the overall win drops to 188.0% (Graphic-5). But the performance is smoother. The Sharpe-Ratio is with 1.17 only slightly lower. The max. relative drawdown on 2011-08-19 is 16.0%.
The 1-month Ant prefers stocks with a higher beta. It gets more conservative over the long run.
Graphic-6 shows the performance of the best Donkey-setting. The momentum is calculated from the last year, but the current month is omitted. The performance is 216.3% with a monthly Sharpe-Ratio of 1.15. The max. relative drawdown is on 2011-08-11 with 17.3%. The performance for a 6-month and a 1-month momentum is slightly worse.
The relation between Ant and Donkey is similar to the Diversified-Portfolio. The Ant has only a slight edge in the overall win. The main difference is the smoother behavior. But the difference is for the Nasdaq-100 less pronounced. The stocks have a high positive correlation structure. Considering the covariance is hence less beneficial.
Note: For this reason the following graphics look (almost) the same

Graphic-4: Ant: Nasdaq-100, 1-Month window (orange), QQQ (yellow) 2009-12-30 till 2014-03-27.
Graphic-5: Ant: Nasdaq-100, 6-Month window (orange), QQQ (yellow) 2009-12-30 till 2014-03-27.

Graphic-6: Donkey: Nasdaq-100, 2-12 Momentum (orange), QQQ (yellow) 2009-12-30 till 2014-03-27.
Improving the Ant with the IVTS (Revision 1):

The Implied Volatility Term Structure is generally defined as:

\[ \text{IVTS}(t) = \frac{\text{Short-Term-IV}(t)}{\text{Long-Term-IV}(t)} \]  

(1)

One selects on this curve points with readily available implied volatility measures. Following [8] the interesting measures for the \textit{Short-Term-IV} are the VXST (9-days IV calculated from weekly options), the VIX (1-month from monthly options) and the VX30. The VX30 is the price of a VIX future with a maturity of 30 calendar days. Usually such a future does not exist. In this case one calculates the weighted mean of the 1\textsuperscript{st} and the 2\textsuperscript{nd} nearest future. If the first future has a maturity of 20 days, and the second a maturity of 50 days, one calculates the VIX futures 30 value as \(2/3 \times \text{future}_1 + 1/3 \times \text{future}_2\). VIX futures have an own inherent logic, but there is obviously a close relation with the implied volatility-surface of SPX options.

For the \textit{Long-Term-IV} the VIX, the VXV (3-months from monthly options), the VX30 and the VX45 are used. The VX45 is the price of a VIX future with 45 days maturity. It is the weighted mean of the 2\textsuperscript{nd} and 3\textsuperscript{rd} future. One can select any maturity, but 30 and 45 days have been the most useful ones in previous studies. The considered combinations are:

\[
\begin{align*}
\text{VXST}(t)/\text{VIX}(t) & \quad (1a) \\
\text{VXST}(t)/\text{VXV}(t) & \quad (1b) \\
\text{VXST}(t)/\text{VX30}(t) & \quad (1c) \\
\text{VIX}(t)/\text{VXV}(t) & \quad (1d) \\
\text{VIX}(t)/\text{VX30}(t) & \quad (1e) \\
\text{VIX}(t)/\text{VX45}(t) & \quad (1f) \\
\text{VX30}(t)/\text{VX45}(t) & \quad (1g)
\end{align*}
\]

In [8] it was shown that smoothing the IVTS with a median-5 filter improves the result considerable. Median filters are popular in image processing to remove salt and pepper noise. The salt- and pepper-noise of trading applications are volatility spikes due to market overreactions. The median-5 removes up to 2 spikes within the last 5 trading days. One avoids short term whipsaws. The downside is that the reaction to a real turning point/crash is delayed. But like in [8] the median-5 filter clearly improves the result. The median-3 filter is in contrast a poor choices. It just delays the reaction.

Graphic-7 shows the effect of the median-5 filter for the IVTS (1e) from 2014-01-01 till 2014-04-02. IVTS (1e), the relation between the VIX and the VX30 is – like in previous studies - the most interesting combination. The median-5 filter does a relative good job of removing the spikes while preserving the overall movement of the IVTS.

The IVTS is a measure for the market regimes the S&P-500. One implicitly assumes that the portfolio follows the same pattern. This is reasonable for a portfolio of Nasdaq-100 stocks. This assumption does – by construction – not hold for the Diversified portfolio (e.g. it wins in Aug. 2011).

The application of the IVTS is straightforward. If the filtered IVTS is rising above a threshold, one steps at the sideline. If the IVTS falls under the threshold one enters the playing-field again. For the simulation the regular re-balances at the end of the month are not changed. If one goes e.g. on 5\textsuperscript{th} September out of the market and enters the market again on 15\textsuperscript{th}, the re-balance at 30\textsuperscript{th} September is nevertheless done. If the IVTS is at this date above the threshold one re-balances at the first day in October the IVTS criterion is met. One wants to keep the regular EOM re-balance pattern.
Graphic-8 shows the result of the unchanged Ant for the Nasdaq-100. This is in principle the same than Graphic-4. The results differ slightly, because trading is extended by a few days and a new month for the Sharpe-Ratio calculation is added. I have also slightly modified the initialization routine of the Differential Evolution. The overall performance is now 224.7%, with a Sharpe-Ratio of 1.14 and a max. relative drawdown of 16.7% at 2011-08-19 (red line in graphic 8).

In Graphic-9 the IVTS (1e) with a stop-loss threshold of 1.10 is used. This limits the loss in August 2011 to 5%. The overall win increases to 249.1% with a Sharpe-Ratio of 1.18. The max. relative drawdown is now 10.9% at 2011-11-25. The threshold is relative high. It reacts only to severe crashes like in August 2011.

Graphic-10 shows the performance if one sets the threshold to 1.05. The performance drops to 213.3% with a Sharpe-Ratio of 1.14. The max. relative drawdown stays exactly the same. But the lower threshold makes a difference end of January 2014 (red line in graphic-10). The higher threshold and the plain Ant loose in this phase 5%. The filtered IVTS never reaches 1.10. The lower threshold pulls the ripcord and limits the losses to 3.4%. But one misses of course also winning streaks. If one wants to play safe the 1.05 threshold is better. If one wants to avoid only a massive downside risk the higher threshold is – at least in the historic simulation – preferable. The result depends of course also on the overall market conditions. There was in the last years a general tailwind. If one stays too long in the save harbor, the other boots pass by.
Graphic-8: Ant: Nasdaq-100, 1-Month window (orange), QQQ (yellow) 2009-12-30 till 2014-04-02.

Graphic-9: Ant-IVTS-1.1: Nasdaq-100, 1-m window (orange), QQQ (yellow) 2009-12-30 till 2014-04-02.
Conclusion:
The Ant is a momentum strategy which takes in a model-free way the covariance structure of the assets into account. It uses the well known – but in the quant literature often ignored - fact that powerful computers are already invented. Instead of a rigorous mathematical model like in MPT the Differential Evolution Optimizer solves the problem. Calculating a portfolio takes 1-2 seconds. The Ant is an attractive alternative to the Donkey or other risk-adjusted momentum strategies. Adding the IVTS smooths and improves the performance further. It is a matter of risk appetite if one prefers a higher or lower threshold.

References: