Riding on the LIBOR and EURIBOR-Forward-Rate-Curve:
The Arkhi-Strategy
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Mongol Arkhi – Light liquor made from Isgelen Tarag or Airag (Kefir from mare-milk). This completely transparent beverage has a good reputation especially among mongolian men, because it was traditionally the strongest drink available. 

Abstract:
This working paper examines the behavior of the Eurodollar-Futures forward-rate. After some theoretical investigations a simple and profitable trading strategy is examined. The strategy relies on the typical monotonic increasing ED-term-structure.

Revision-1 contains only minor improvements. The changes are – like this sentence - in the font Tahoma-10.

Revision-2 contains a dynamic method to adjust the volume of the hedge. This improves the performance slightly. The changes are – like this sentence - in the font Constantia-10.

Revision-3 extends the model to Euribor futures. The original font Times New Roman is used. A complete new part is added at the end of the previous paper.

Revision-4 extends the model to Short-Sterling futures. A complete new part is added at the end of the previous paper.

Introduction:
Eurodollar-Futures are since many years the most liquid futures contract. They are cash-settled on the LIBOR-Forward-Rate. They main series is quarterly, running up to 10 years. At the moment EDZ12 till EDU22 is available. The expiry date is the Monday before the third Wednesday (this is not the same than the 3rd Monday). The notational is 1.000.000 US-$. But the relevant payoff is the difference to the LIBOR at the expiry date. Each base-point (1/100th of a percent) is worth 25 US-$. The ED-Futures are quoted as 100-LIBOR-Forward-Rate in %. If the Forward-Rate for EDZ13 is 1.5%, EDZ13 would be quoted as 98.50. A detailed description of ED-Futures can be found in [1] or on the CME-homepage.

Graphic-1 shows the price of EDZ12, EDZ13 and EDZ14 between 2010.01.01 and 2012.10.26 (the available time-series ends at this date. All results will hence be reported till 2012.10.26).
The futures have a clear positive-term structure. The price of EDZ12 is always above EDZ13 which in turn trades always higher than EDZ14. Or in other words, the Forward-Rate is always monotonic increasing. The difference between the futures decreases as time goes by. Due to the positive-term structure there is an overall upwards price-trend. But this does not mean, that the price is always going up. There was a relative large decline between Nov. 2010 and Feb. 2011. The decline is almost parallel. Or in other words there was a parallel shift of the forward-rate-curve.

Note: Usually one considers in Fixed-Income-Finance the yield-curve. There is a direct relationship between these two curves. The yield-curve is the integral over the forward-rate. Yields are for bonds and treasuries more handy, because they measure the overall performance of a bond/treasury. But ED-Futures are contracts on the forward-rate. It makes no sense to transform the forward-rate into the yield.

Graphic-1: EDZ12, EDZ13, EDZ14 from 2010.01.01 till 2012.10.26
(Source: www.godotfinance.com)

The Forward-Rate Term-structure:

Graphic-2 shows the forward-rate term-structure at 2010.10.26 (orange), 2011.04.26 (yellow), 2011.10.25 (green), 2012.04.24 (blue) and 2012.10.26 (magenta).
The y-axis is the time to maturity (the forward-time) in years. The x-axis is the forward-rate in % (the futures price is 100 – rate). One sees that for all curves the forward-rate increases with time. The slope is leveling off for longer maturities at the far right. The slope is from 2010.10.26 to 2011.04.26 increasing (futures prices were falling) and is falling from this date on.

This curve does NOT mean, that the traders expected in Apr. 2011 a LIBOR of 5% in 2018 (7 years on the x-axis). Like for almost all futures the price is determined by arbitrage with replicating portfolios. The 5% just say, that 7 year fixed-income LIBOR-instruments have a yield of somewhat less than 5% (if the yield-curve is increasing, the forward rate is always above the yield). The forward-rate decreases till Oct. 2012, because the yield of mid-term investments decreased in the last 2 years.

Note: Up to my knowledge only VIX-futures prices are expectations (plus a risk-premium). There is no way to replicate VIX-Futures and hence the price is not determined by arbitrage.

Graphic-2: Term-structure of forward-rates between 2010.10.26 and 2012.10.26

The term-structure looked in the pre-2008 period different. The FED has clamped since 2008 the short-time interest rate to (almost) Zero. The term-structure can only increase. Before 2008 the typical shape was like in Graphic-3. There was a hump at the left. The short-term LIBOR rate was around 5%. The forward-rate declined first by about 0.5% before climbing up again. The overall level was much higher, but the slope (after the hump) was less than currently.
Graphic-3: Term-structure of forward-rates at 2006.10.16 and 2007.10.26

Graphic-4 shows the first-differences, the slope, of the term-structure. The dates and colors are the same than in Graphic-2. In April 2011 (yellow) the sloped was highest at about 1.5 years. Or with other words, EDZ12 and EDU12 had the largest forward-rate difference of all consecutive futures. Since that time the maximum has shifted up to 3 to 4 years. The range with the largest slope is the best place for the forward-rate ride. One can notice in the last 2 years a drift of the maximum slop to the right. At 2012.10.26 the maximum was at 4 years. Hence it would be best to enter at this date the EDZ16 contract.
Graphic-4: First difference (slope) of term-structure 2010.10.26 till 2012.10.26

**Modeling the Term-Structure:**

There are numerous approaches to model the term-structure with a parsimonious model. One approach are splines. Splines are a general method (they were initially developed for automobile construction). Every reasonable well behaved function can be approximated by splines. But splines are usually not a very parsimonious representation. Another popular method is the Nelson-Siegel model. The complete term structure is determined by 4 parameters. For the forward-rates the function is defined as

\[ f(t) = b_0 + b_1 \exp(-t/\tau) + b_2 \frac{t}{\tau} \exp(-t/\tau) \]

The parameters have an intuitive meaning. For long-term futures the forward-rate approaches b0. The second parameter b1 is the (negative) slope of the curve. The third parameter b2 describes the curvature. One could call b2 also the hump parameter. The parameter tau is the time dilation. For a small tau, the \( \exp(-t/\tau) \) term declines very fast to zero. For a larger tau, the influence of b1 and b2 is present over the whole curve. For pre-2008 term-structures the fit is almost perfect (graphic-5).

The blue model-curve has like the empirical yellow line a hump at the beginning. The slope is over the whole curve quite similar. In case of the green model-curve and the orange empirical the fit is even better. It is impressive that one can fit a non-linear

and humped curve with 4 parameters. The result is less impressive for the post-2008 curves. The yellow model-curve has still a hump at the left, although the empirical curve is monotonic increasing. In the mid and long-term the fit is again close to perfect. It does not help to remove the hump-term b2. The hump disappears, but the fit of the slope deteriorates.

Diebold and Li use in [2] the Nelson-Siegel model to forecast the whole term-structure. The parameters b0, b1, b2 have the additional nice characteristic of being almost independent. One can forecast them with 3 independent autoregressive models. But this is not the case for tau. Tau relates b1 and b2 to each other. Therefore Diebold and Li calculate the mean-tau of several term-structures and clamp in all models and in the forecasts tau to this value. This works in their paper. It definitely does not work here. Tau varies for the term-structures over a wide range. This is not only the case if one compares pre- and post-2008 curves. Tau changes even for relative nearby curves. Fixing tau to the mean (or any other value) results in rather poor approximations. The approach is at least for Eurodollars useless. There are several other forecasting approaches around. But none of them looked convincing. It is obviously as difficult to predict the term-structure than to predict other assets prices. The term-structure forecast is even more demanding, because one predicts in parallel 40 futures prices. I have therefore not followed this approach further.
The Arkhi-Strategy:

Arkhi is a mare-milk liquor. This sounds from a gastronomic point of view not very attractive. Probably the best of Arkhi is: It's in Mongolia the hardest drink around. One gets drunk. But it seems to be the right drink for a strategy which rides on the forward-curve. As said above, one can't directly forecast the forward rate. But one has tailwind from the positive slope of the curve. One can implement basically a carry-rollover-strategy. The tailwind is largest around 3 years. It is almost zero at the far end and since the FED-clamp also for the first few maturities.

A logical approach is therefore to go e.g. in 2010.10.26 the EDH14 long. To keep this position for a year till 2011.10.26 and to roll over to EDH15. One keeps EDH15 for another year and rolls then over (in this case, the end of the time-series is reached).

The basic Arkhi strategy does exactly this. The calculation assumes an initial account of 500.000 US-$. One goes 60 futures contracts long. These numbers are arbitrary. The volume is for simplicity not adjusted to the available account. One does not reduce the position after losses and does not increase after a win. One just keeps the 60 futures. Adjusting the volume would increase the performance/win of the strategy.

Graphic-7 shows the result for the strategy if one starts trading at 2010.10.26 (till the end of the time-series at 2012.10.26). This was the worst starting point in the post-2008 regime. The whole forward-curve was rising till March 2011 and the strategy incurs severe losses. But in the end one wins 312,000 $. The yellow curve is the hedged-strategy. For the hedge one calculates a short 10 trading days (2 weeks) and a longer 84-trading days (4-months) arithmetic moving average. Whenever the short MA is under the long MA, one hedges by selling -60 the next nearby futures. If one is the EDH14 long, the hedge is the EDZ13. During an active hedge the risk (and win) is reduced to the movement of the spread between EDZ13 and EDH14. The hedge reduces the drawdown of the basic Arkhi considerable. At the end both strategies give practically the same result. The result is rather robust in the parameters of the MA. One can even use for the short-MA the current price. One can also use a long MA of e.g. 100-days. It does not really matter.

Graphic-8 shows the same strategies but for the last 4 years. The drawdown between Nov. 2010 and March 2011 is of course the same. There is also a significant drawdown in June 2009. But one is this time never under water. The hedge (yellow-line) works in the June-2009 drawdown quite well. The decline was at that time sharp but shorter than in Nov. 2011. At the end both strategies are again practically the same. They almost triple the initial cash to 1,450,000. Obviously the much smoother hedged-version is preferable.
Instead of hedging with a nearby future one can also go out of the market if the short-MA is below the long-MA. During the hedge only the small spread is traded. The two strategies are therefore as expected quite similar (graphic-9). As the performance is practically identical, one would prefer the simpler go to the sideline approach.

I have tried also with more distant hedges. E.g. Hedging EDH14 with EDH13. But this just increases the drawdown. One could increase the hedging ratio. E.g. hedging with -90 instead of -60 EDH13. This does not improve the situation, because the forward-curve shifted parallel upwards.

Instead of using a short- and long-MA I have tried also other indicators: The popular Relative-Strength-Index of Wellers did not work at all. A simple momentum indicator (hedge if the price was falling in the last N-days) performed better than RSI, but did not improve over the MA approach.

**Hedging with a Dynamic-Quantity:**

The Hedging so far used a static hedge quantity. 60 long was hedged with -60 short. In this approach one calculates for the daily-price changes the following equation.

\[ \Delta \text{Long}(t) = a_0 + a_1 \times \Delta \text{Hedge}(t) \]

The hedging volume is then defined as
\[ \text{hedge-qty} = -a_1 \times \text{qty} \]

If one hedges with more nearby futures, \( a_1 \) is \( \geq 1.0 \). More nearby features have smaller daily changes than futures with larger maturity (at least in the considered range of the term-structure). The hedge-quantity is usually – in absolute terms – somewhat larger.

The equation is recalculated at each hedging-day (if the hedging-signal is not set, the calculation makes no sense). The regression uses the last quarter (last 63-trading days) for the calculation. This recalculation is done, because the relation between the long- and the hedging-future changes when one moves along the forward-curve. It can also change due to different market conditions.

The regression parameters are calculated with the Theil-Sen estimator. The Theil-Sen estimator is in contrast to OLS a robust estimator with a number of nice statistical properties (see \([4]\)). The dynamic hedge improves the result slightly, but it is overall no real boost. For hedging with the most nearby future \( a_1 \) is close to Zero. The hedge-quantity does not differ too much from the constant volume. The results deteriorate again for more nearby futures. The hedge does work properly, because the ups- and downs are not anymore in lockstep.

Note: This paragraph was added in Revision-2:

![Graphic-8a: Hedging with constant volume (orange) and dynamic-volume (yellow) from 2010.10.26 till 2012.10.26](image)

The other extreme is to hedge all the time. One trades a calendar spread. This does not improve the result. It decreases in comparison to the no-hedge strategy the drawdown, but it reduces the win in the same way. One can generate the same effect by reducing the leverage aka volume.

Instead of watching the game from the sideline, one could go short once the short-MA falls under the long-MA. This is usually done in stock trading strategies. But it is for ED-
Futures an uphill battle. The whole strategy depends on the assumption of an increasing term-structure. Instead of a tailwind, the short position has a significant headwind. As can be seen in graphic-10 going short increases only the volatility. But it does not improve the overall performance.


The S&P VIX-Futures Indices (see [3]) are similar in spirit. These indices roll-over continuously. For the short-term index, one rolls on a daily basis the most nearby over to the next future. In the mean the distance to maturity is always 30 days. In the Arkhi-Strategy the distance to maturity differs up to one year. One rides on difference places of the curve. I have also implemented the VIX-methodology. One starts with 60 EDH14 and rolls either each week, month or quarter over to EDH15. I could not find any improvement. It's only more complicated. The slope is in a wide range similar. The exact point on the curve does not matter.

Note: The continuous roll-over is the reason for setting the volume to 60. One gets for the monthly and quarterly roll always a rollover-quantity of 5 or 15 futures.

Instead of doing a complete roll once a year, one could also roll-over semiannually or quarterly (from EDH14 to EDU14 in case of a semiannual approach). It does – for the same reason as described above – not really matter. The yearly roll-over plus going on the sideline once the short-MA moves under the long-MA is the most simple approach.
For the considered time-range an initial maturity of 3.25 years was overall best. But as already noted above, the point of the maximum slope has move forward. For a new trade an initial maturity of 4 years would be probably better. One would enter in Nov. 2012 an EDZ16.

The trading frequency is even with the hedge very modest. Graphic-11 shows the relation between the EDH14, the short MA(10) and the long-MA(84) from 2010.10.26 till 2011.10.26. The EDH14 is in this time-period the active future (other futures behave similar, because they move in parallel). There are 4 crossovers. For EDH15 the number of crossovers are just 2 (not shown).
Euribor-Futures (added in Revision-3):
Euribor Futures are conceptually identical to the Eurodollar. The underlying is the Euribor. It's the Interbank offered rate of Eurozone banks. Currently the behavior of Eurodollar and Euribor Futures are similar. The Euribor-Futures (ticker EB) traded at the CME are somewhat less liquid than established Eurodollars (ED). This is especially true for longer maturities. But for maturities which are interesting for the Arkhi strategy liquidity is sufficient.

Graphic-12 and 13 show the Euribor and Eurodollar Term-structure in the last year. Both term-structures flatten (which is bad for the Arkhi). The Euribor is slightly steeper than the Eurodollar. The slope is from 2.5 to 4 years almost linear. The buckle at the right are probably liquidity effects.

The slightly higher slope of the Euribor increases also the profit. Graphic-14 and 15 compare the performance of EB and ED in the last two years. The final result of the EB is 937.250$ and for ED 854.000$ (all the parameters are the same as described above).

The Euribor should be traded with slightly shorter maturity. One buys with a maturity of 3 years and rolls over after one year. For the considered time-range one buys initially EMH15 and rolls at 2011.11.30 to EMH16. At 2012.11.30 the next roll is to EMH17.

Note: One does not enter EMZ14, because 10 trading-days before expiry one rolls over.

For the full 2 years an initial maturity of 2.5 years (starting with EMU14) is slightly better. But for the last year the longer maturity of 3 years is superior. The same effect can be seen for Eurodollars. Shorter maturities flatten more than longer ones. One has therefore to move towards the longer end of the term-structure.
Graphic-12: Euribor-Term-Structure 2011.11.30 till 2012.11.30

Graphic-13: Eurodollar-Term-Structure 2011.11.30 till 2012.11.30
Graphic-14: Euribor Arkhi from 2010.12.04 till 2012.12.03

The Short-Sterling Futures are conceptually identical to the Eurodollar. The underlying is based on the BBA-LIBOR for three month sterling deposits. The unit of trading are 500,000 £ (for the Eurodollar it is 1,000,000$). The expiry is the 3rd Wednesday in a quarterly month. The symbol is “L”. This differs slightly from the Eurodollar- and Euribor-Futures which expiry on the Monday before the 3rd Wednesday. But the different expiry is irrelevant for the Arkhi-Strategy.

Graphic-16 shows the change of the term-structure between 2008.12.05 till 2012.12.05. In 2008 the term-structure had a typical humped-shape. In 2009 the Bank of England clamped the short rate down, but the longer expiries did not go down immediately. This happened step by step in the following years. The term-structure is now quite flat. For curiosity graphic-17 shows the fit of the Nelson-Siegel forward-curve model for the humped 2008 curve. The model was obviously constructed for this type of curve. The fit is almost perfect.

Graphic-16: Short-Sterling Term-Structure at 2008.12.05 (orange), 2009 (yellow), 2010 (green), 2011 (blue), 2012 (violet).
Graphic-17: Short-Sterling Term-Structure at 2008.12.05 (orange) and Nelson-Siegel model (yellow).

As the Short-Sterling Futures are traded in 500.000 £, the initial value of the Arkhi-Strategy is set to 310.000 £. The volume is 75. This corresponds approx. to the 500.000 starting value and the volume of 60 for the Eurodollar-futures. Graphic-18 shows the performance from 2010.12.05 till 2012.12.06. The final value for the best strategy is 487.000 £. This is approx. 784.000 $. The Short-Sterling futures deliver the lowest profit. This profit is mainly generated from April to October 2011. The best strategy would trade at 2010.12.05 LH14. But the Arkhi stays on the sideline, because the MA(10) is below the MA(84). The strategy avoids the losses in this drawdown. It looses somewhat in the following short peak, but fully gains the profits in the recovery from April till October (Graphic-19). Since Oct. 2011 the wins are modest. The term-structure of the Short-Sterling has already become too flat.
Graphic-18: Performance of Short-Sterling Arkhi from 2010.12.06 till 2012.12.07

Graphic-19: LH16 from 2010.12.06 till 2011.12.10 (orange). Yellow-line is MA(10), Green is MA(84).
Conclusion:
The Arkhi- is for the current form of the forward-rate-curve a very simple and profitable strategy with low trading-costs. There is – like for any other strategy – no free lunch. One depends on the mercy of the FED to clamp the short-term rate close to Zero. But even a moderate increase of this rate (by 0.25% or 0.50%) would be no disaster. The increase on the 3-years curve should be less. Or with other words, the curve should flatten. At least the curve was much flatter in the pre-2008 regime. But even if the FED sticks to the Zero-clamp, the curve can become – like in Nov. 2010 - steeper. This would generate losses for the strategy. But the losses should be limited by the hedge (by going out of the market). At reentry a steeper curve would be even more profitable. So far the curve has flattened in the last 1.5 years. The effect can be studied for the US-Treasury ETN Flattener (FLAT) and the US-Treasury ETN Steepener (STPP). The FLAT has won in the last 1.5 years, the STPP lost. These ETN's trade the US-Treasury yield-curve. This is not the same than the LIBOR-forward-rate, but there is a high correlation between these two curves. In short: The risk for the Arkhi seems to be acceptable.

Eurodollar or Euribor? (added in Rev. 3)
The behavior is very similar. The slope of the Euribor is slightly larger. The volatility of both series is comparable. At the moment the EB has a slight edge over ED. But it should be noted, that the Euribor is influenced by the policy of the European Central Bank. The behavior of the ECB is less predictable/consistent than the policy of the Fed. ECB's policy is a complicated compromise of very different national interests. The Fed strategy seems to be more consistent and predictable.

Short-Sterling? (added in Rev. 4)
The Short-Sterling term-structure has flattened even more than the Eurodollar and the Euribor. The wins are especially in the last year modest. The only reason to trade additionally L-Futures would be diversification. But the general behavior is at the moment very similar. There seems to be little benefit to put another egg in the Short-Sterling basket.

References: