Approximating the Implied Volatility of SPX-Options with the VIX.

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*amare et sapere **vix** deo conceditur*

Both Wisdom and Love are **barely** granted, even to the Gods.

Abstract:

Data-collections of SPX options are not for free and sometimes also cumbersome to handle if one wants to be up-to-date. This working paper shows that the implied volatility and hence the price of SPX-options can be easily approximated with the VIX. The approximation works reasonable well for backtesting simple short- and mid-term SPX-option strategies.

Introduction:

For backtesting an SPX-options strategy one can buy data e.g. from DeltaNeutral. Besides the price-tag it is not trivial to clean, process and update the data. I was asked by the Sibyl-Fund-Trader Siddharth Bhatia if one can calculate the information with the freely available and easy to handle VIX-data.
The VIX measures the 30-days implied volatility of SPX-options. The calculation (of the new index) is not based on the Black-Scholes model. It is a model-free approach over the whole range of actively traded strikes. Usually there are no options with a maturity of exactly 30 days available. The VIX-calculation uses the weighted mean of the two most nearby monthly options. The VXST Short-Term Volatility Index uses the same methodology, but has a mean maturity of 9 days. It uses the 2 most nearby Weekly options. The VXV is the 3-months (90 days) equivalent of the VIX.

The IV of an option depends on the volatility-smile. OTM-Puts have a higher IV than ATM-Puts. OTM-Calls have the lowest. The simplest approach is to calculate for each strike-factor or each delta the regression.

$$IV_{Kf}(t) = b_0+b_1*VX(t) \qquad (1)$$

or

$$IV_{delta}(t) = b_0+b_1*VX(t) \qquad (2)$$

Where the strike-factor aka moneyness is defined as

$$Kf = K/SPX \qquad (3)$$

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VX = VXST for maturity (in calendar days) <= 9.
VX = w*VIX + (1.0-w)*VXST for 9 < maturity <= 30
w = (maturity-9)/21
VX= w*VXV + (1.0-w)*VIX for 30 < maturity <= 90
w = (maturity-30)/60
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The VX is a maturity-weighted mean of VXST, VIX and VXV. Up to 9 days one uses the VXST, from 9 to 30 days the weighted mean of VXST and the VIX, between 31 and 90 days the weighted mean of the VIX and the VXV are used.

Using the VX instead of the plain VIX improves the fit of the linear-regression considerable. One takes the effect of the maturity on the IV-surface better into account.
The fits of regressions (1) and (2) are reasonable. One can improve the approximation somewhat with the following regression:

$$IV_{Kf}(t) = b_0+b_1*VX(t)+b_2*VX^{\wedge 2}(t)+b_3*maturity(t) \quad (3)$$

or

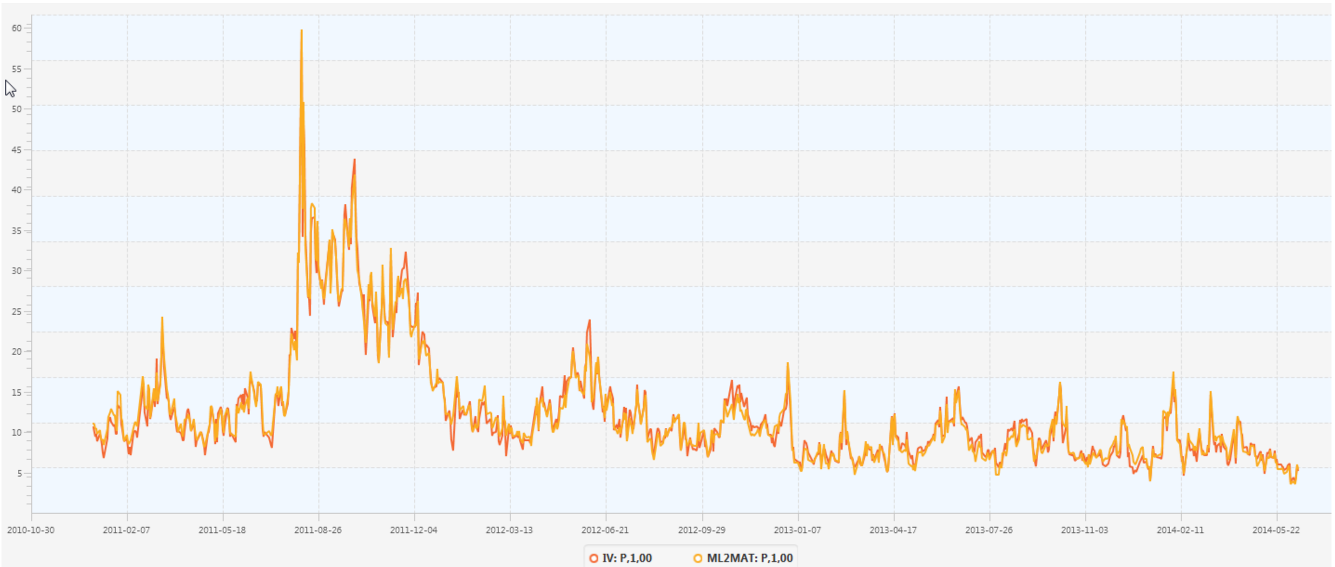
$$IV_{delta}(t) = b_0+b_1*VX(t)+b_2*VX^{\wedge 2}(t)+b_3*maturity(t) \quad (4)$$

I tried other equations, but (3) and (4) gave the best fit.
Instead of the strike-factor Kf one can use also the delta of the option. But for calculating delta one needs the unknown implied volatility. One can handle this by first calculating the IV with (3). One plugs this value into Black-Scholes formula and uses the resulting delta to get the corresponding parameters of (4). One can of course always stick to the IV_{Kf}. My observation is: The in-sample fit of (4) is better than (3). But the result is less clear out of sample.

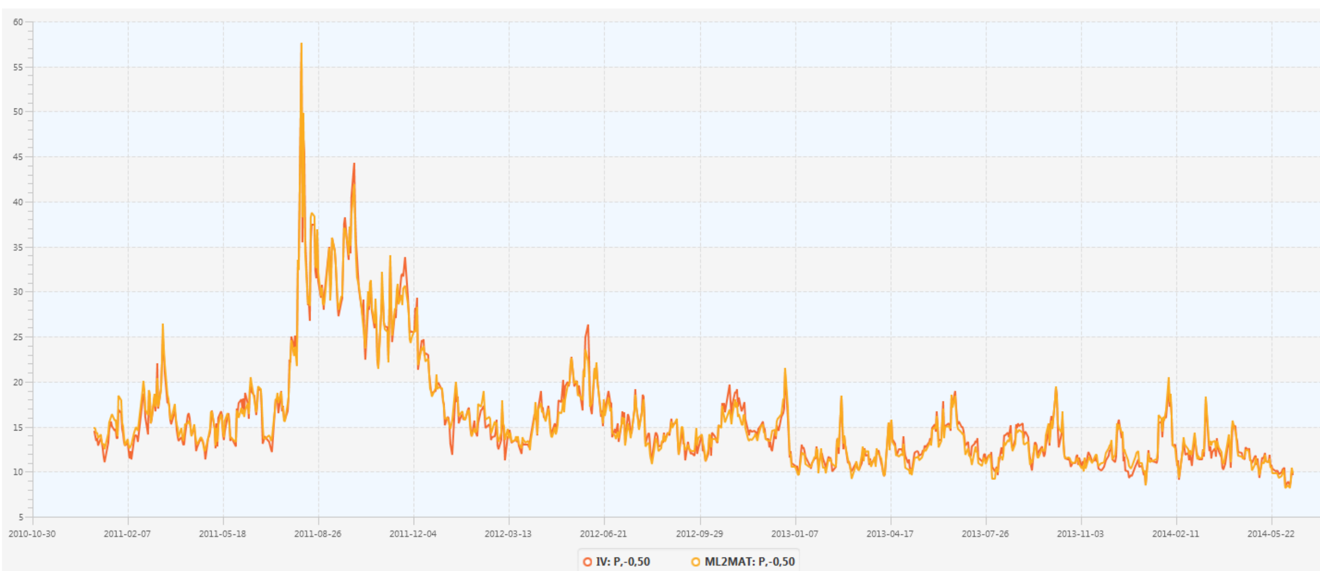
The Data and the Calculations:

For calculating the regression parameters I used daily SPX-options data from DeltaNeutral from 2011-01-01 to 2014-06-14.
For calculating the parameters of Table-I (the tables can be downloaded in *.csv format from the links given at the end of this paper) I ran the following procedure: The most nearby monthly SPX-option nearest to the given strike-factor or delta is selected. The option is rolled over if the maturity (in calendar-days) is less than 7. The maturity is between 7 and 35 or 42 calendar days. One rolls on the Monday of the 3rd-Friday week to the next expiry. The calculation does NOT use at each day the same strike. One holds the Kf or the delta fixed. The strike moves with the SPX.
For Table-II the procedure was similar but the roll was done already 20 days before the expiry. This simulates a trading strategy where one buys/sells also the 2nd expiry. There is some overlap in the maturity. The estimated results are not identical, because the parameters are estimated from different data.
Table-III is suited for a short-term-options strategy. The calculation uses weekly options. One rolls on Thursday to the next-Friday option. As before, there is some overlap with Table-I. But the estimation is done from a quite different data sample.
All the calculations ignore interest. The interest-rate parameter in the Black-Scholes formula is set to Zero. The effect is for options with a short maturity minor. The interest was also in the estimation period practically Zero.
To estimate the IV for a given option one calculates first the strike-factor Kf and the maturity in calendar days. One looks up in the tables the row with the closest Kf and calculates from (3) the IV. One can already stop at this point. One plugs in the IV into the Black-Scholes equation and gets the approximated delta and price of the option. Alternatively one can use the delta to look up the row with the nearest delta. One calculates with the parameters in the matching row with (4) the IV. The delta-approximation is – as already noted – in-sample somewhat better. But I have done also out-of-sample calculations in the current very low-volatility regime. Sometimes the direct calculation with the Kf, sometimes the result from the iterative delta-step was closer to the market-value. There was no clear winner. One could iterate over the delta-step. But there are usually no significant changes in the IV anymore. The approximation estimates basically the mean-slope of the smile. The slope depends on the market regime. There is no clear cut way how to model this relation. The quadratic VIX-term in (3) and (4) is a first approximation of this dependency. The slope of the smile depends also on the maturity. This is handled by the maturity term. But these are only crude proxies.

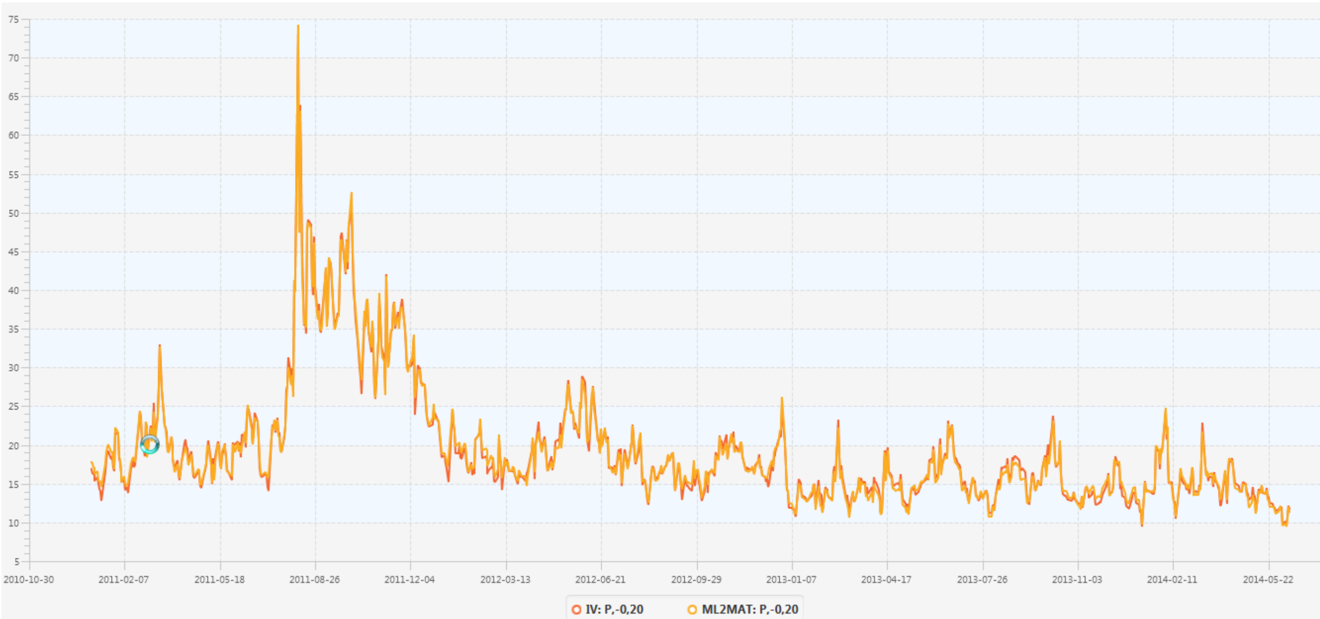
The following graphs show the in-sample fit between the measured IV (red) and the estimated IV (yellow) according to equation (3) or (4) from 2011-01-01 till 2014-06-14.



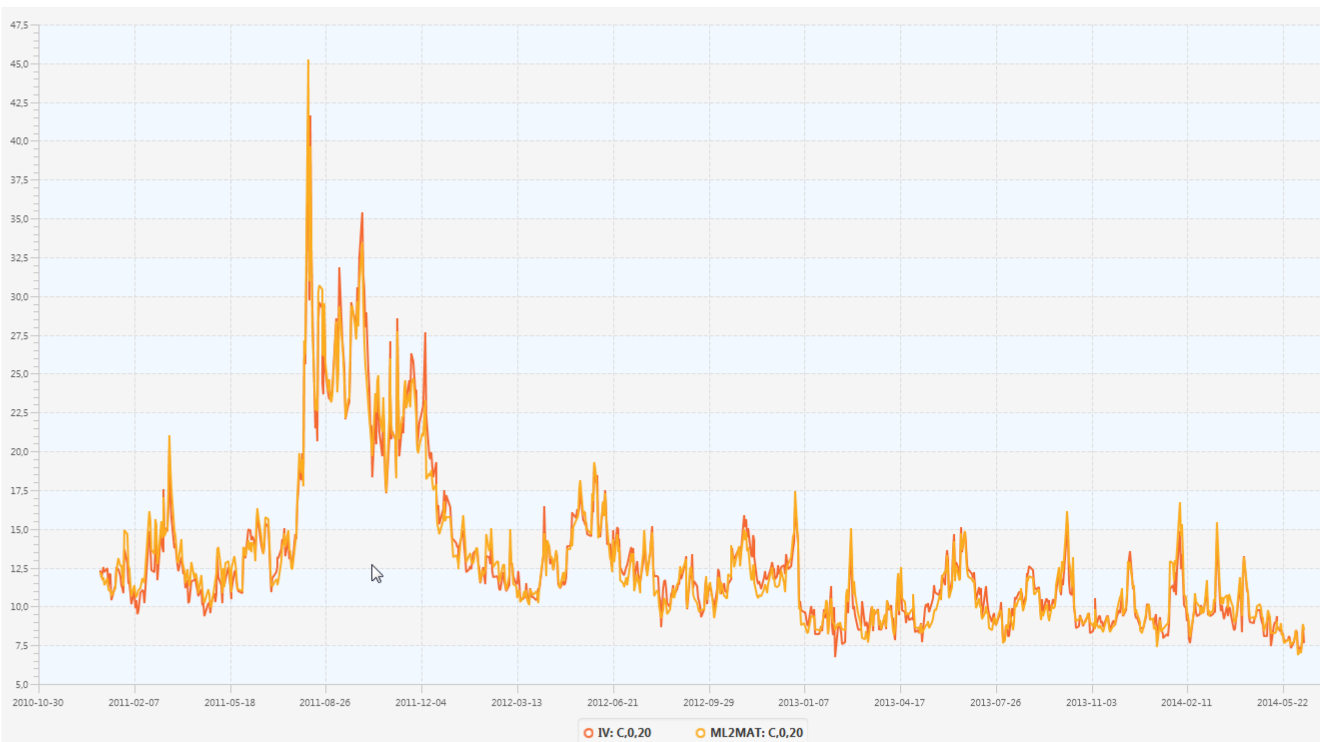
Graphic-1: Strike-Table-I, ATM Put. Measured IV (red), Estimated IV (yellow).



Graphic-2: Delta-Table-I, ATM Put. Measured IV (red), Estimated IV (yellow).



Graphic-3: Delta-Table-I, OTM Put, Measured IV (red), Estimated IV (yellow).



Graphic-3: Delta-Table-I, OTM Call, Measured IV (red), estimated IV (yellow).

Conclusion:

The data for the approximation are free and one has always the latest VXST-, VIX- and VXV-index values available. The calculation is straightforward and at least as fast than looking up the values in the DeltaNeutral database. One avoids the problem of impossible or very unrealistic option quotes. This is quite a nasty detail in any data-collection I have processed so far. It is not at all a trivial task to clean the data and to remove unrealistic quotes. The backtested strategy will exploit such unlikely prices. One

will never get this extra profit in real-time trading.
The approximation values are always within realistic bounds. The approximated data may have a bias, but they are never completely wrong. One can interpret the approximation as a sort of noise-filter.
One approximates the mean-slope of the smile. The extra terms b_2 and b_3 do not fully correct for the movement of the slope in different market regimes. If the strategy is based on exploiting special slope conditions, the approximation is not well suited. The approximation error is – according to in- and out-of-sample tests - within 0.01 (or 1%). Instead of the measured IV of 0.15 one gets an approximation between 0.14 and 0.16. Usually the approximation is within a bound of +/- 0.05, but the deviation is larger in very low and very high volatility regimes.

Links for Downloading the Tables in csv-Format:

Delta-Indexed Table-I (delta_VixVix2Mat_7.csv)
Strike-Indexed Table-I (strike_VixVix2Mat_7.csv)
Delta-Indexed Table-II (delta_VixVix2Mat_20.csv)
Strike-Indexed Table-II (strike_VixVix2Mat_20.csv)
Delta-Indexed Table-III (delta_VixVix2Mat_2.csv)
Striked-Indexed Table-III (strike_VixVix2Mat_2.csv)