The dynamic hedging of weekly S&P-500-Futures Options.  
Cashbot case studies.  
Chrilly Donninger  
Chief Scientist, Sibyl-Project  
Revision 1: 2016-02-09  
Revision 2: 2016-02-29

Abstract:
Cashbot is a fully autonomous trading bot. I have added recently S&P-500 weekly Options trading. This is a hot topic with a lot of risk and fun. The strategy is currently tested with paper-trading. This note analyzes a dynamically hedged short strangle. This note does not give definite answers how to handle such a position in a complicated market situation. But it addresses several important practical questions. 
Revision 1 introduces a new Hedging method. The study extends the results to the next time period. There is hence no hindsight bias. 
Revision 2 extends the results to the next time period from 2016-02-09 to 2016-02-19.

The General Setting:
Cashbot is a fully autonomous stand-alone trading bot. It is programmed in JavaFX and implements the Interactive-Broker API. One has direct access to the market over the IB-Gateway. I have implemented in the past several VIX-Futures and VIX-ETN strategies (see [1] and the references herein). The latest addition to the Cashbot toolbox are short S&P-500 Futures weekly Options strategies. The VIX-Strategies are traded for real money. The Weekly-Options are currently in the paper-trading testing phase. Weekly Options are from a speculative point of view very interesting. They have a higher implied volatility and a fast time-decay. The underlying ES-Futures are traded – besides a short break at 3:15 CST – around the clock. The Futures are even overnight liquid and have a minimal bid-ask spread of 0.05 pts (2.5$). This is of course not for free. Dynamic hedging is especially in the last phase a difficult task. If something goes wrong on the final Thursday or Friday it is practically not possible to close the position. There is also a marked implied volatility skew.

The Position:
I entered at Friday 2016-01-08 at 14:50:36 UCT a short strangle with -10 ESF6 P1900 and -10 ESF6 C1925. The Options expire one week later at 2016-01-15 22:00:00 UCT or 4:00 P.M. CST. There is physical delivery into the underlying Futures.  
Note: All times are - if not explicitly noted - UCT.
The following analysis uses historic data downloaded from Interactive-Brokers. The data have a time-resolution of 5 minutes. Cashbot checks in the standard setting every 10 seconds the real-time market prices and adjusts – if necessary – the hedge. The historic data end 5 minutes before the expiry at 21:55 (The market closed at 22:00 and the historic data-services of IB reports only traded prices). The historic analysis is hence not completely identical to real-trading. But the differences are minor and do not change the overall picture.  
The ESH6 noted at 2016-01-08 14:50:00 at 1945.50. The P1900 had a (mid-)price of 10.00, the C1975 of 12.50. The ESH6 fell almost 100 points to 1850.5 on 2016-01-15 17:45. The price of the P1900 went up by 500% to 50.0. But one can not really speak of an Options market-price. The ESH6 was 5 minutes before at 1852.5 and the P1900 had a “price” of 43.00. The Put was hence cheaper than the intrinsic value of 47.5. This is not an artifact of the historic data. It happened also in real trading live. The reason
is a very wide bid/ask spread. The quoted prices are mid-prices. The mid-prices bounce around almost randomly. This has immediate consequences for the hedging process. The calculation of the Implied-Volatility uses also mid-prices. The mid-price is in general much more reliable than the last trade-price. If the Options price is smaller than the intrinsic value one can't compute the Implied-Volatility. The IV is in turn needed for the Delta-calculation. If the price bounces up one gets in connection with the small remaining time absurdly high IV values of up to 1000%. This has also consequences for the delta-calculation. Although one is deep in the money with almost no time left Delta does not approach -1.0.
The random bid-ask bounce is for the real-time calculation more severe than for the historic simulation. The real-time sampling frequency is 30 times higher (the period is 10 to 300 secs). The chance to get complete garbage increases considerable. One can also not close the position anymore. The market is even for such a small position not liquid enough and one would have to pay the absurd high bid prices. It is never fun to sit on an illiquid position which runs in the wrong direction. But the pain is modest if one is properly hedged. It is a nightmare for a naked short position. The final value of the ESH6 at 21:55 was 1876.50 with a Put price of 23.50. The real settlement price at 22:00:00 was 1875.0. ES-Futures have a physical delivery. If the position is not fully hedged one gets assigned ESH6 Futures at 1900 (the Put strike). This assignment is somewhat nasty because one can't trade the Futures anymore. The market is closed. There are also Options on the SPX-Index available. The SPX Options are cash-settled. But one would have to hedge with the same Future. The SPX and ES-Options have practically the same Implied-Volatility (for the same/similar moneyness). The downside of SPX-Options is the larger overnight gap. The fact that the underlying and the hedging instrument differ is for short term Options no real issue. The SPX and ES-Futures trade in a fixed and predictable relation.

Graphic-4 shows the Delta of the short Strangle. The yellow line is the usual BSM-calculation. The vertical lines on the far right are a result of the – sometimes – negative time-value of the Options. There is no reasonable way to calculate Delta. I set the value to zero to mark this situation. Cashbot ignores currently such situations. The hedge is never changed. But this was one of the points I wanted to address in the historic simulation. I tried several variants. The first one was to use the most nearby VIX-Future as a proxy for the IV. This did not work properly, because the IV of an Option shortly before expiry is also in normal situations much higher than the VIX. The best solution I have found was to use the latest valid IV. The bid-ask bounce can generate also absurd high IVs. I got – in the historic simulation – the best results if the IV is restricted to 120%. The ceiling depends of course also on the instrument and the market-conditions. An IV of 120% is for VIX-Options a typical value and not an upper bound. But for ES-Options it seems to be – under this market conditions - a reasonable clamp.
The red-line is the skew-adjusted Delta. There are numerous methods to adjust for the typical skew of index Options. Sophisticated methods are presented in [2] and [3]. Although theoretically appealing this approaches suffer from the fact that the estimation of the necessary parameters (e.g. vega and the skew) is not at all straightforward (see Graphic-4). If one multiplies the estimated values one can get very unstable adjustment factors. The method is in [2] hence only applied to the weekly rehedge of mid-term ATM Options. The red-line in Graphic-4 uses instead a much simpler adjustment scheme from [4]. The BSM-Delta is adjusted by a fixed factor. The factor depends on the delta. The values are given in table-1 below. If a Put has a BSM-Delta of -0.2, the value is multiplied by 1.25. The adjusted Delta is -0.25. If a Call has a Delta of 0.2, the adjustment factor is 0.75. The adjusted Delta is 0.15. Cashbot uses in the standard setting the Hull&White adjustment. The adjusted value perform also in the historic simulation consistently better than the plain BSM-values. I have watched the performance of CashBot during real-time-trading. The hedge performs locally quite well. The value of the hedged-position does not change significantly if the ESH6 moves up or down a few points. The main lesson of 15 years of computer-chess development is: KISS. This is the only way to guarantee the robust performance of a fully autonomous system. The adjustment factors of Hull&White are certainly not for all market-conditions optimal. But it is also difficult to make something wrong.
In all empirical studies I am aware of dynamic hedging is either done on a daily or even weekly basis. The reason for this choice is probably the lack of HF-Data. It is more reasonable to hedge according the delta of the position. Daily hedging at the close would be for a weekly Strangle a disaster. It is for weekly Options probably better to do no hedging at all. The classical work for determining the optimal hedging strategy is [5]. But this is only a theoretical result. The solution is too involved. E. Whalley and P. Willmot propose in [6] a first approximation to this result. But the formulas depend on very restrictive assumptions on the transaction costs and the price distribution of the underlying. The transaction costs are for ES-Futures minor (and are not considered in this simulation). A much more serious question is the price distribution.

I have implemented a much simpler approach. The position is rehedged, once the delta of the overall position (Options, plus hedging Futures) is outside a given band. This band was set for the paper trade to 1.5 Delta. There are two rehedge approaches possible. In the first case one minimizes the resulting delta. If the Delta is 1.6, one would short 2 Futures to get an adjusted Delta of -0.4. Or one can minimize the number of Futures to get into the band. In this case one sells only 1 Futures and gets an adjusted Delta of 0.6. The second approach is according to Whalley&Wilmott the better choice. It is currently implemented in Cashbot. Rehedging to the middle was in the historic simulation superior.

Setting a fixed band of 1.5 has one adverse effect. Even if the Put is deep in the money the hedge is never set to -10 Futures. The same happens of course also on the upper side. But the Calls were this time not part of the game. I noticed of course this effect. But I did not change the program logic. This is an old habit of mine from many years of Computer-Chess development. Never change the program logic during a game. Analyze instead the problem afterwards and test the new logic in the next game. A consequence is that one is assigned with 1 Future. It was this time almost for free. I could sell the Future at 1897.5. But it is simply a bug of the strategy.

One solution is to implement special logic for the border cases. If the Delta moves above 9.5 adjust the hedge fully. The other solution is to use a conic band. The second approach is more general and also alongside the results of Whaley&Wilmott. If delta is around zero, the band can be wider. If it is near the Max or Min the band gets much smaller.

Graphic-5 shows the performance of the different strategies. Red is the constant bandwidth. Yellow with the special boundary rule. Green the conic-band approach. The difference is visually hard to spot, because the performance is most of the time identical. The strategies differ significantly in the

Graphic-5: Performance with minDelta 1.5 from 2016-01-08 14:50:00 to 2016-01-15 21:55:00
Friday-”endgame”. The fixed-band approach has a final win of 195$, the special boundary-rule improves to 962$. The difference is essentially the cost for not fully hedging the Puts. The conic approach improves to a win of 1437$

The main-reason for the spike down on the right is not a very-sharp move of the ESH6. The ESH6 moved down but the main-effect is what I call the random bid-ask bounce. The Options mid-price was for a short time extremely high.

Graphic-6 shows the performance of the fixed-band approach of 1.5. The red line uses a skew-adjusted Delta, the yellow line plain BSM. The skew-adjustment performs over the whole time-range superior. I have tested this for two additional Strangles. The first one is -10 P1875, -10 C1875 and the second one -10 P1925, -10 C1950. The positions perform generally quite different. The skew-adjusted approach is nevertheless consistently superior.

Graphic-6: Performance with and without skew-adjustment.

Delta-Hedging is not the only game in town. An at the first glance attractive hedging strategy is what J. Hulls calls in [7] the stop-loss strategy. One fully hedges if the Option is in the money and does not hedge at all if the Option is out of the money. This means for the Strangle: Sell 10 ESH6 Futures once the price falls below 1900 and buy them back if the price moves above. The opposite operation would be done if the ESH6 moves above 1975. But it never happened in the considered time range. Another strategy is not to hedge at all. Red is the skew-adjusted Delta-Hedge with a conic bandwidth. The final win is 1437$. Yellow is the stop-loss strategy. The ESH6 moved already at Monday 11th several times around the 1900 strike. Dynamic Delta-Hedging of a short position (negative Gamma) means always “buy high, sell low”. The stop-loss strategy generates each time a considerable loss. One could argue that the situation gets worse due to the 5 minute granularity of the historic data. But a finer time-resolution can turn a bad strategy into a disaster. The loss tends in the limit to infinite (see [7]). The final loss is -7225$. The naked position (green) shows wild up and down swings and ends with a modest loss of -525$.
The situation is different if one replaces the P1900 with the more OTM P1875 (Graphic-8). The Delta-Hedged position has with a win of 832$ a similar performance as before. The overall picture is essentially the same than in the actually traded Strangle. The stop-loss strategy has this time a small win of 212$. But the performance can not be called a hedged position. The naked position swings again around wildly. At the end the full premium of both Options is cashed in. The ESH6 finishes at the strike of the Put. But it is not for the fainthearted to watch the ESH6 falling down to 1850 and having a naked Put position with a strike of 1875 in the Portfolio.

Graphic-9 shows the performance of a strangle with -10 P1925, -10 C1950. The position is initially on both sides close to the money (but not ATM). The stop-loss strategy (yellow) performs here best. The final loss is -1012$. The Delta-Hedged position (red) ends with a loss of -3187$ and the naked position is this time at the end deep in the money. The final loss is -6400$.

One explanation for the loss of the hedged position is: The underlying ESH6 has in the considered time range a HF-Realized Volatility of 29.37%. Actually the RV is 4.15%. The 29.37% is the scaled up yearly value. The OTM Puts have at inception a higher IV than the RV. The more ATM a lower. But the result is also path-dependent and can not simply explained by the difference of IV to RV. Generally a higher IV at the end hurts more than at the beginning (see [8])
Besides the short market break of 15 minutes at 21:15 (3:15 p.m. CST) hedging was so far done around the clock. Graphic-10 shows the effects if one stops hedging between 21:15 and 08:00 (2:00 a.m. CST). The extended market hours of SPX-Options start at this time. One can practically hedge the SPX-Options by using the IV of the most nearby equivalent ES-Option. E.g. the ES is at of this writing at 1828.75, the SPX at 1835.97. One can calculate with good approximation the IV of the SPX P1800 with the ES P1795. Also the more liquid ES P1800 should work with a small smile-correction factor. The difference of the ES to the SPX is within a small band which changes only slowly towards the expiration of the ES-Future. Hedging of SPX-Options has anyway to be done with the ES-Futures. But it is nevertheless an interesting exercise to test if hedging around the clock is worth the money. For the position at hand the answer is no: The yellow line is the performance of the hedge-position with the gap between 21:15 and 08:00 the following day. The final value is 2612\$ to 1437\$. The red-line is not smoother than the yellow one. The overnight-gap strategy avoided one sell and buy back. And it changed the hedge 2 times a little bit later. Avoiding one symmetric transaction is clearly an advantage. The two cases were the adjustment was done somewhat later were also no disadvantage. This must and
will not always be the case. There was this time relative little action overnight and a lot of action during the day. Ignoring the overnight ups- and downs is in such a situation beneficial.

The situation changes (at the last trading day) if one restricts dynamic hedging to the regular trading hours from 14:30 to 21:15 (green line). The regular trading hours strategy has a final loss of -2625$. It lost 5000$ in the 3 hours before the regular opening at Friday 15th.

Graphic-10: Hedging 24h and with different overnight gaps.

Graphic-11 shows the effect of different bandwidths. The red line is the standard-setting with 1.5 and a final value of 1437$. The yellow line uses a very small band of 0.5 (1125$). A 2.5 Delta band has the highest final value of 1662$ (green). But the swings in between are much larger than for the tighter bands. A wide band of 3.5 (blue) has a lower final value (1112$) and additional ups- and downs. The standard setting of 1.5 seems to be a reasonable choice.

Graphic-12 shows the effect if does only the minimal rehedge into the bound (red) line or if one rehedges to minimize the resulting Delta (yellow). The bandwidth was set to 2.5 to get a marked
difference. But the picture is for other bandwidths the same. According to [6] is rehedging to the boundary the preferable approach. According this case study one should rehedge to the middle. One case study is of course no conclusive result. But the question seems to be less clear than suggested by the literature.

Conclusion:
The considered market conditions were a good test for a short Strangle and probably also a lot of other short strategies. The stop-loss-strategy can work in certain conditions. But it is not robust and reliable. Robustness is the main criterion for a fully autonomous trading strategy. The inferiority of stop-loss is a well know result which was confirmed also in this example. The skew-adjustment of Hull&White is very simple to implement. It is a very promising concept. The conic bandwidth which guarantees that deep in the money Options are fully hedged seems to be also an improvement over a fixed width band. In any case one has to guarantee that in the money Options are finally fully hedged. Otherwise one gets - beside extra losses - for ES-Options the physical assignment problem.
If one wants to avoid the nasty problems of the last Friday one has to close the position already on Wednesday. In this case one has to enter not the nearest, but the second or third weekly. This Options are also quite liquid. There are of course a myriad of other short strategies. The Strangle is a relative simple strategy with a high profit potential. The considered time range was not a worst case scenario. But it was at least an interesting week.
Revision 1:
**Hedging without Futures:**
An alternative to buying/selling Futures is to readjust the Options position. There are numerous possibilities for this approach. If the ES-Futures are falling, one could pull down the Call strike by buying back the initial strike and selling a new position with a lower strike. This approach is advocated in [10]. One could also buy additional Puts with a lower strike. The most trivial action is to buy back some of the initial written Puts. After such an adjustment the position would consist of e.g. -10 Calls, -8 Puts. If the Futures fall further, one buys back another tranche. If the Futures bounce up, one buys back Calls. The overall position is reduced in each step. The strategy is more conservative than moving the strikes up- and down. The number of transaction is also limited. Hedging is done only into the border of the area. The bandwidth is set to 1.0 Delta. Delta is calculated with the skew-adjustment.

Graphic-13: Hedging by adjusting the Options. Bandwidth 1.0 Delta

The red line in Graphic-13 shows the performance of the P1900, C1975 Strangle. The strategy works in this case quite well. There is only a minimal drawdown and the final win is 1835$. The OTM P1875, C1975 (yellow) combinations does less well. There is less initial premium and the buyback is as expensive, because the Puts are bought back when the Futures have moved down further. The final loss is -1025$. The green line shows the more ATM P1925, C1950 Strangle. The position is already closed in the initial swings of the Futures. There is little risk and no fun. If one sets the bandwidth to 1.5 Delta, the performance of the P1900, C1975 Strangle drops to 400$, but the other two Strangles are improved (Graphic-14). Setting the bandwidth to 1.25 Delta gives the best overall result. The P1900,C1975 Strangle has a win of 975$, the P1875,C1975 is break even and P1925,C1950 wins 547$. But one can probably always find a bandwidth which performs well.
The new position:
I entered at Monday 2016-01-25 at 12:30 UCT a short strangle with -10 EW1G6 P1825 and -10 EW1G6 C1940. The Options expiry at 2016-02-05 at 22:00:00 UCT or 4:00 p.m. CST. The EW1G6 are not the nearest but the second weekly series. I wanted to exploit the effects of a somewhat longer time span. The ESH6 Future was at 1896.25. It closed at 1873.25 at the expiry. Both options were over the whole time range out of the money. But the ESH6 made between this range several sharp swings (Graphic-16). A naked Strangle without any hedging was hence very profitable. One cashed in the full premium of 11325$. The naked Strangle was never in the negative area. (Graphic 17). The performance of the stop loss strategy is identical.
The sharp moves of the ESH6 were a severe problem for Delta Hedging. One looses at each move a considerable amount of money. The red line in Graphic-18 shows the performance a 1.5 Delta bandwidth. Hedging is done only to the interior and not to the middle of the band. One looses -2475$. Extending the bandwidth to 2.0 Delta improves the result to a small win of 287$ (yellow line). A bandwidth of 2.5 Delta performs with plus 3237$ best. Hedging to the mean is much worse. The sharp
swings cost in this case the double amount of money.
The Delta-Skew-Adjustment was clearly superior in the previous trade. This is for this sample not the case. Graphic-19 shows the performance with plain BSM-Deltas. The performance is -2075$ for a 1.5 Delta Bandwidth, 900$ for 2.0 Delta and 4300$ for 2.5 Delta. Hedging to the mean is also for the plain BSM-Delta clearly worse.

Hedging by adjusting/buying back the Options position performs satisfactory. Graphic-20 shows the performance for a bandwidth of 1.0 Delta (red). The P&L is 2897$. There is no loss over the whole time range. A band of 1.25 Delta (yellow) works slightly better. The P&L is 3195$. The 1.5 Delta bandwidth is with 2795$ somewhat worse. Delta is calculated with Skew-Adjustment. Like for the Futures Hedging the performance is improved if one uses the plain BSM-Delta and avoids the adjustment. The 1.0 Delta bandwidth has a P&L of 3145$ (red line in Graphic-21). The 1.25 Delta improves to 3717$ (yellow) and the 1.5 Delta bandwidth is this time best with 4277$ (green).
Graphic-21: Plain-Delta with Options Strangle EW1G6 P1825, C1940 with different Bandwidths

Graphic-22 shows the effects if one stops hedging between 21:15 and 08:00 (2:00 a.m CST) with Skew-Adjusted Delta. The performance is for a bandwidth of 1.5 Delta (red) with -2787$ slightly worse than hedging around the clock. It is for the 2.0 Delta bandwidth better and for 2.5 Delta again per un pugno di dollari worse. The sharp swings happened this time within the active trading period. There was little overnight action. There is also for the Options-Hedging strategy little difference between hedging around the clock and ignoring the overnight movements.

Graphic-23 shows the performance of an ATM Strangle with a P1890 and C1900. Delta-Hedging with the Futures results in a considerable loss of -10577$ for the 1.5 Delta bandwidth, -7937$ for 2.0 Delta and -8300$ for 2.5 Delta. There is no significant difference between the skew-adjusted and the plain BSM Delta calculation. The skew-adjustment has only for OTM options an effect. The naked Strangle makes a profit of 17775$ (blue line). But it is at the beginning with -7150$ in the red zone. One would probably trigger a stop-loss at this point.
Hedging by buying back options is also for the ATM Strangle a relative conservative strategy. There is relative little risk but also not too much fun. The 1.0 Delta bandwidth has a P&L of 1337$ and a max. Loss of -812$ (red in Graphic-24), the 1.25 Delta bandwidth a P&L of 1897$ and a max. Loss of -2212$ (yellow), the 1.5 Delta bandwidth a P&L of 2197$ and a max. Loss of -2325$ (green).

The overall picture is for a (far) OTM Strangle P1800, C1960 similar (Graphic-25). Delta-Hedging with skew-adjusted Delta results in a (considerable) loss. The naked Strangle cashes in the full premium of 6825$. But it has also a max. Loss of -1825$. The plain BSM-Delta performs somewhat better (Graphic-26)

The similar pattern for the different Strangles is no big surprise. The realized volatility of the ESH6 was higher than the implied volatility. But as already noted it was the swing of a pendulum and not a directional movement. Delta-Hedging suffers from these swings, the naked positions are fine at the end, but would probably trigger in between a stop-loss signal.
Hedging by buying back options works also for the OTM Strangle relative well. Graphic-27 shows the performance with the plain BSM Delta calculation. The 1.0 Delta bandwidth has a P&L of 2555$ and a max. Loss of -1150$ (red), the 1.25 Delta bandwidth a P&L of 2070$ and -1500$ max. Loss (yellow). The 1.5 Delta Bandwidth a P&L of 3910$ and max. Loss of -1475$ (green). The skew-adjusted Delta performs somewhat worse, but the overall pattern is the same.
Conclusion for Revision-1:
The skew-adjustment of Hull&White was for the first sample a clear improvement over the plain BSM-Delta. For the new trade it is the other way round. The performance is as expected determined by the movement of the Futures and not by the moneyness of the Strangle. There are numerous methods in the literature for forecasting the realized volatility. One can think about to avoid the written Strangle if the model forecasts are unfavorable. Or turn the trade around and invest in a long Straddle. Hedging by buying back options seems to be an interesting conservative strategy. Another quite different alternative is the naked Strangle.
Revision-1 addresses – as expected - several new questions and gives no definite answers.

Further Work for Revision-1:
Evaluate different volatility forecast models. The models should use High-Frequency data. The measure is in the first step the prediction quality. It is well known that one can predict the realized volatility. But the real interesting measure is the economic value. It is less clear if one can improve a trading strategy by these forecasts.

Revision 2 - The new position:
I entered at Monday 2016-02-09 at 15:00 UCT a short strangle with -10 ESG6 P1800 and -10 ESG6 C1900. The Options expiry at 2016-02-19 at 22:00:00 UCT or 4:00 p.m. CST. The ESG6 are the monthly 3rd Friday options and in this context the second weekly series. The time to expiry is similar to Revision 1. The ESH6 Futures were at the entry at 1851.0 and at the expiry at 1914.25 (Graphic-28). I had this time for the simulation only HF Data with a delta T of 15 min. available. The results are nevertheless close to the Real-Time results.
Note: One could store Real-Time data during trading. This feature is currently not implemented. Instead I use the Historic-Data API from Interactive Brokers. The amount of data is rather restricted.
Graphic-28: ESH6 between 2016-02-09 15:00 and 2016-02-19 at 22:00:00

Graphic-29 shows the performance of the ESG6 Strangle. The red line is the naked position. The Call is at the end in the money. But the combined premium is higher and there is a final P&L of 6350$. The yellow line shows the Delta-Hedged position with a bandwidth of 1.5 Delta. The Delta is Skew-Adjusted. The final P&L is 2137$. The green line uses the plain BSM Delta. The final performance is with 3025$ somewhat better. But skew-adjustment was superior at the beginning when the S&P moved down. The blue lines show the performance of a 2.0 Delta bandwidth. Light-blue uses the skew-adjustment, dark-blue the plain BSM Delta.

Graphic-30 shows the result for the Options-Hedging strategy. If the S&P moves down, one buys back Puts, if it moves up, one buys back Calls. The red line uses a bandwidth of 1.0 and a skew-adjusted Delta. The final P&L is -692$. The yellow line uses the same bandwidth, but Delta is not adjusted. It has a minor loss of -82$. The green line uses a wider bandwidth of 1.5 and skew-adjustment. The P&L is 290$. The blue line uses a plain BSM-delta and has a P&L of 1325$. The plain BSM-Delta is like for the Futures hedge the better choice. The Futures-hedge is also the better strategy. The S&P moved first down. The strategy buys back a part of the Puts at a higher price. Then it moves up. The Puts get far
OTM with practically no Delta. So in the next step the Calls are bought back. The overall action is financed by the premium of the remaining Puts. But there is little to gain. The Futures hedge profits from the declining volatility in the upwards move. As the S&P moves almost up in a straight line, the hedges are profitable. The picture would be different if the S&P would move down again in the last few days.

Mid-Term-Trading:
Before the advent of weekly options the maturity of typical option-trades was (at least) 4-5 weeks. The positions in Revision-1 and 2 split this up in 2 different entries. The following simulations analyze the performance of the strategies under a mid-term time frame. A strangle with P1800 and C1950 of 3rd Friday ESG6 Options is entered at 2016-01-19 at 15:00. The starting time is determined by the restrictions of the Historic-Data API of Interactive Brokers. The ESH6 Futures started at 1887.5, climbed up to 1939.0 at 2016-02-01 20:15, declined to 1807.0 at 2011-02-11 11:45, moved up again to 1930.0 at 2016-02-18 12:00 and finished with 1914.25 at the expiry. The start- and final-values are relative close to each other, but there were severe swings in between (see Graphic-31).
Graphic-32: Futures Hedge for P1800/C1950 Strangle.

Graphic-32 shows the performance of the naked and the Futures hedged positions. The naked position has larger swings. But it is almost all in the win-area and cashes in the full premium of 14825$. The swings of the Futures hedged positions are of course much smaller. But the large movements of the underlying eat up the premium. With a bandwidth of 1.5 the skew-adjusted Delta has a final P&L of -5300$ (yellow), the plain BSM Delta behaves in between different but has with -5237 almost the same final P&L (green). A bandwidth of 2.0 Delta improves the performance to 987$ for the skew-adjusted Delta (light-blue) and to -937$ for the plain BSM Delta (dark blue).

Graphic-33: Option Hedge for P1800/C1950 Strangle.

Selling back the options works somewhat better. Red and yellow show the performance of a 1.0 Delta bandwidth with skew-adjusted (red) and with plain BSM Delta (yellow). The P&L is 925$ and 1362$. A bandwidth of 1.5 performs better (green and blue line). This time the skew-adjustment (green) outperforms the plain BSM Delta (blue) with 3205$ to 2457$.

The Put and the Call have at the entry a Delta of approx. +/- 0.25. One can enter a more OTM position.
The overall picture is for the P1775/C1975 similar. The naked position (red) cashes in the full premium of 9575$. It is never in the loss area. The 2.0 bandwidth (dark- and light-blue) is again superior to the 1.5 bandwidth. All delta-hedged strategies loose money. The worst performance of -5212$ has the 1.5 bandwidth with plain BSM Delta (green).

The option-hedged strategy performs also for the more OTM strangle superior. The 1.5 bandwidth (light- and dark-blue) outperforms the 1.0 band (red, yellow). Plain BSM is better then the skew-adjustment (yellow to red, dark- to light-blue). The performance is between 1232$ (red) and 2850$ (dark-blue).

Graphic-36 shows the performance of an (almost) ATM strangle with P1875 and C1900. For the naked position the Call is at the end somewhat in the money. But the premium is much larger and the final win is 27725$. The worst is the plain BSM delta with a bandwidth of 1.5 (green). It looses -6287$. The best is the plain BSM delta with a bandwidth of 2.0. It makes a small win of 400$. The P&L of the options hedge is around break-even (Graphic-37). This time the smaller bandwidth of 1.0 is superior to the larger one.
The Iron-Condor:

M. Benklifa presents in [11] an Iron-Condor strategy. The short Puts and Calls are far OTM. The strikes of the protective long positions are 25 points away. All strikes are at 0 mod 25. The volume is for these strikes higher. The analyzed Iron-Condor is hence the far OTM Strangle with P1775/C1975 of above plus the long Put at P1750 and the long Call at C2000.

Graphic-38 shows the performance. Red is the naked strategy. One cashes in the full premium of 3750$. The futures hedges with a bandwidth of 1.5 and 2.0 Delta behave exactly the same, because the position Delta stays within the band. The yellow and green line show a bandwidth of 1.0. Yellow with skew-adjustment, green with plain BSM. The strategies are initially the same, the position delta stays within the 1.0 band. The skew-adjusted Delta is finally a loss, the plain Delta a small win. But according to [11] one does not delta hedge the position. M. Benklifa proposes to close the position if it has lost 2/3 of its initial value (won 2/3 of the full premium). There is also a stop-loss. One buys back a wing if the underlying moves too close to the short strike. He quantifies “too close” in terms of points. A more reasonable measure is the delta (these type of books avoid delta, because amateur readers are not comfortable with Black-Scholes). A reasonable choice for delta seems to be 0.4. Graphic-39 shows the implications of these rules. The position is already closed at 2016-02-08 9:45 with a P&L of 2575$. One avoids indeed a lot of fuss in the following days. The premature stop-win opens also the possibility
to enter – with similar margins – a new forward rolled position. The stop-win rule is not necessarily less profitable than keeping the position till maturity (there are of course additional trading costs). One can run also into troubles in the rolled-over position. The delta-stop has for this position no effect. Neither the short Call nor the short Put have a delta which is absolutely larger than 0.4.

The situation changes if one builds from the initial Strangle P1800/C1950 an Iron-Condor. If one lets the position run one gets a P&L of 5250$ (red line in Graphic-40). The taking the profits premature has for this Condor no immediate advantage. The position is closed in the upwards moving phase of the ES-Futures at 2016-02-12 17:45:00. The P&L is 3500$ (yellow). If one closes a wing with a delta-threshold of 0.4 one gets for the skew-adjusted Delta the performance of the green line. The Put-Wing is closed at 2016-02-08 16:00. The final P&L is 1350$. With the plain BSM Delta the Call-Wing is closed before at 2016-01-29 20:45:and the Put-Wing at 2016-02-11 08:30. The final P&L is -3500$.

One could think about to close both wings together to avoid swing-high, swing-low losses. This limits the loss to -1425$ in the second case. But it also reduces the win of the first case to 1000$.
The Iron-Condors of [11] have a somewhat longer maturity of 5 to 6 weeks. Additionally they are not written on ES-Futures but on SPX Index Options. The index Options are European and cash settled. This is for an options-writing strategy certainly an advantage. The disadvantage are the restricted trading hours. This is for an Iron-Condor less critical than for a Strangle. The max. loss of both considered positions is 12500$.

**Conclusion for Revision-2:**
Hedging with futures was for the short-term position clearly superior to buying back the options. After the first 3 days the S&P was moving up and realized volatility declined. This favors the Futures hedge. There is no similar advantage for buying back the options. The plain BSM Delta was also in Revision 2 slightly superior to the skew-adjusted values. A mid-term position over the whole time-range of Rev. 1 and 2 suffers – as expected – similar problems than the combination of the 2 short-term trades. The Condor is a relative conservative alternative. The rules of M. Benklifa seem to be reasonable. But it should be noted that the CBOE Iron-Condor-Index CNDR (see [12]) has a poor performance. The rules of the CNDR are considerable different to the ones presented in [11].

**Further Work for Revision-2:**
*Evaluate different volatility forecast models. The models should use High-Frequency data (Rev. 1).* Some work has been done in the meantime, but there are yet no really convincing results. Keep on working.

**References:**